MCV4U Name: SOLUTIONS

Vectors Unit Test

Expectation	Level Achieved
C2 - perform operations on vectors in two-space and three-space, and use the properties of these operations to solve problems, including those arising from real-world applications	
C4 - represent lines and planes using scalar, vector, and parametric equations, and solve problems involving distances and intersections	

Thinking More About Communication:

	Incomplete I	Unacceptable R	Poor 1	Acceptable 2	Good 3	Outstanding 4
TECHNICAL CORRECTNESS OF SOLUTIONS	All or most solutions are blank	No solutions are correct or many left blank	Few solutions are technically correct	Some solutions are technically correct	Most solutions are technically correct	All or almost all solutions are technically correct
PRESENTATION OF SOLUTIONS	All or most solutions are blank	No evidence of presentation or many solutions left blank	Solutions to few problems stand alone	Solutions to some problems can stand alone	Solutions to most problems can stand alone	Solutions to all or almost all problems can stand alone

Expectation C2

1. Given $\bar{a} = [-4,9]$ and $\bar{b} = [-3,1]$, determine;

a) $\vec{a} + \vec{b}$

$= (-7, 10)$

b) $\left|\vec{b}\right|$

$= \sqrt{(-3)^2 + 1^2}$

$= \sqrt{10}$

c) $2\bar{a} - 3\bar{b}$

$= 2(-4,9) - 3(-3,1)$

$= (-8,18) - (-9,3)$

$= (1,15)$

d) Write $\vec{a}$ using the unit vectors $\vec{i}$ and $\vec{j}$

$\vec{a} = -4\hat{i} + 9\hat{j}$

e) $\bar{a} \cdot 0$

$= \varnothing$

2. If $\vec{a} = [-3,2,5]$ and $\vec{b} = [-6,-4,-10]$, determine:

a) $\vec{ab}$

$= (-6-(-3), -4-2, -10-5)$

$= (-3, -6, -15)$

b) a vector perpendicular to both $\vec{a}$ and $\vec{b}$

$$\vec{a} \times \vec{b} = \begin{array}{cccccc} -3 & 2 & 5 & -3 & 2 & 5 \\ -6 & -4 & -10 & -6 & -4 & -10 \end{array}$$

$= (-20-(-20), -30-30, 12-(-12))$

$= (0, -60, 24)$

c) the angle between $\vec{a}$ and $\vec{b}$

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

$$= \frac{(-3)(-6)+(2)(-4)+5(-10)}{\left(\sqrt{(-3)^2+2^2+5^2}\right)\left(\sqrt{(-6)^2+(-4)^2+(-10)^2}\right)}$$

$$= \frac{-40}{(\sqrt{38})(2\sqrt{38})}$$

$\theta = 122^\circ$

d) $3\vec{a} \cdot \left(2\vec{a} - 5\vec{b}\right)$

$3(-3,2,5) \cdot (2(-3,2,5) - 5(-6,-4,-10))$

$= (-9,6,15) \cdot (24,24,60)$

$= -216 + 144 + 900$

$= 828$

Collinear

3. Determine if the vectors $\vec{AB} = [-1,-4,1]$ and $\vec{AC} = [-4,-1,4]$ and $\vec{AD} = [2,-7,-2]$ are coplanar.

coplanar

$m(-1,-4,1) + n(-4,-1,4) = (2,-7,-2)$

$-m - 4n = 2$ ①

$-4m - n = -7$ ②

$m + 4n = -2$ ③

① $m = -4n - 2$

② $-4(-4n-2) - n = -7$

$16n + 8 - n = -7$

$15n = -15$

$n = -1$

① $m = -4(-1) - 2$

$= 4 - 2 = 2$

Check:

③ LS

$m + 4n$

$= 2 + 4(-1)$

$= 2 - 4$

$= -2$

RS

-2

LS = RS

$\therefore$ vectors are coplanar.

4. Determine the value of k such that $\vec{u}$ and $\vec{v}$ are perpendicular. Given $\vec{u} = [k, 5]$ and $\vec{v} = [-3, 1]$

$u \cdot v = 0$

$-3k + 5 = 0$

$-3k = -5$

$k = \frac{-5}{-3}$

$k = \frac{5}{3}$

5. Calculate the area of the triangle with vertices at the following points:

P(4 , -2 , -3) , Q(2 , -1 , -1) and R(0 , 2 , 3)

$\overline{PQ} = (2-4, -1-(-2), -1-(-3)) = (-2, 1, 2)$

$\overline{PR} = (0-4, 2-(-2), 3-(-3)) = (-4, 4, 6)$

$\overline{PQ} \times \overline{PR} = \begin{array}{cccccc} -2 & 1 & 2 & -2 & 1 & 2 \\ -4 & 4 & 6 & -4 & 4 & 6 \end{array}$

$= (6-8, -8-(-12), -8-(-4))$

$= (-2, 4, -4)$

$|\overline{PQ} \times \overline{PR}| = \sqrt{(-2)^2 + 4^2 + (-4)^2}$

$= \sqrt{4 + 16 + 16}$

$= \sqrt{36}$

$= 6$

Area of triangle:

$\frac{1}{2} |\overline{PQ} \times \overline{PR}|$

$= \frac{1}{2}(6)$

$= 3$

3 units^2

Expectation C4

6. Suppose line L passes through A (1 , 2 , -3) and B (3 , 2 , -6). Express the equation for L in each of the following forms;

a) vector

$\vec{m} = (2, 0, 3)$

$\vec{r} = (1, 2, -3) + t(2, 0, 3)$

b) parametric

$x = 1 + 2t$

$y = 2$

$z = -3 + 3t$

c) symmetric

Not possible.

$\frac{x-1}{2} = \frac{z+3}{3}, \; y = 2$

7. Find the scalar equation of the plane that passes through the points A (4 , -5 , 1) , B (2 , 3 , 3) and C (0 , 2 , -4)

need 2 direction vectors:

$\overline{AB} = (-2, 8, 2) = \vec{a}$

$\overline{BC} = (-2, -1, -7) = \vec{b}$

cross product to find $\vec{n}$

$\vec{n} = (-2, 8, 2) \times (-2, -1, -7)$

$= \begin{matrix} \cancel{-2} & 8 & 2 & -2 & 8 & \cancel{2} \\ \cancel{-2} & -1 & -7 & -2 & -1 & \cancel{-7} \end{matrix}$

$= (-56 - (-2), -4 - (14), 2 - (-16))$

$= (-54, -18, 18)$

$\therefore \pi : -54x - 18y + 18z + D = 0$

Sub in any point to find D:

$-54(0) - 18(2) + 18(-4) + D = 0$

$-36 - 72 + D = 0$

$D = 36 + 72$

$= 108$

$\therefore$

$\pi : -54x - 18y + 18z + 108 = 0$

8. Write vector and parametric equations for the plane which contains the points A (1 , 2 , -3) , B (5 , 1 , 0) and C (3 , 2 , -6).

need 2 direction vectors:

$\overline{AB} = (4, -1, 3)$

$\overline{AC} = (2, 0, -3)$

Vector form:

$\vec{r} = \vec{r}_0 + t\vec{m} + s\vec{n}$

$\vec{r} = (1, 2, -3) + t(4, -1, 3) + s(2, 0, -3)$

* there are many other combinations.

Parametric form:

$x = 1 + 4t + 2s$

$y = 2 - t$

$z = -3 + 3t - 3s$